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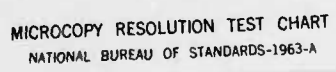
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PLANE STRESS CRACK-LINE FIELDS FOR CRACK GROWTH  
IN AN ELASTIC PERFECTLY-PLASTIC MATERIAL

by

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ABSTRACT: Mode-I crack growth in an elastic perfectly-plastic material under conditions of generalized plane stress has been investigated. In the plastic loading zone, near the plane of the crack, the stresses and strains have been expanded in powers of the distance,  $y$ , to the crack line. Substitution of the expansions in the equilibrium equations, the yield condition and the constitutive equations yields a system of simple ordinary differential equations for the coefficients of the expansions. This system is solvable if it is assumed that the cleavage stress is uniform on the crack line. By matching the relevant stress components and particle velocities to the dominant terms of appropriate elastic fields at the elastic-plastic boundary, a complete solution has been obtained for  $\epsilon_y$  in the plane of the crack. The solution depends on crack-line position and time, and applies from the propagating crack tip up to the moving elastic-plastic boundary. Numerical results are presented for the edge crack geometry. ←

*Epsilon sub y*

KEY WORDS: crack propagation, Mode-I, elastic-perfectly-plastic behavior, strain on crack line.



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### Introduction

Quasi-static fields of stress and deformation near the tip of a growing crack in an elastic perfectly-plastic material, have been discussed in considerable detail by Rice [1]. Analytical expressions for the near-tip fields reveal the asymptotic structure of the fields in the immediate vicinity of a moving crack tip. They contain, however, functions that can generally be obtained only by supplemental numerical procedures. The one complete analytical solution, which is for the case of anti-plane strain, has been given by Rice [2].

In a recent paper Achenbach and Dunayevsky [3] considered the case of Mode-I crack growth under plane stress conditions in an elastic perfectly-plastic material. In Ref.[3] it was assumed that the stress components for a centered fan field, which were discussed by Hutchinson [4], and which satisfy the yield condition and the equilibrium equations, are valid up to the elastic-plastic boundary (at least near the crack line). The analytical approach of Ref.[3] then employs expansions of the particle velocities in powers of  $y$  (the distance from the plane of the crack), to obtain ordinary differential equations with respect to  $x$  for the coefficients in the expansions. Functions of time that enter in integrating these equations were determined by matching the fields in the plastic loading zone to the dominant terms of suitable elastic fields at the elastic-plastic boundary.

In the present paper we reconsider the results of Ref.[3]. Instead of making an a-priori assumption on the stress field, we also write expansions for the stresses near the crack line. Substitution of these expansions into the equilibrium equations and the yield condition produces a simple system of equations for the coefficients. Unfortunately the system is not closed, and additional information is required. The structure of the equations suggests that the cleavage stress on the crack line is uniform. With that assumption, the system of equations can be solved in a simple manner. The resulting expressions for the stresses are consistent with the centered fan field employed in Ref.[3].

To illustrate the analytical results the plastic strains just ahead of a moving crack tip have been computed for an edge crack geometry. The results are particularly suited for use in conjunction with a critical strain criterion.

### Governing Equations

The geometry that is being considered in this paper is shown in Fig. 1. The  $x_3$ -axis of a stationary coordinate system is parallel to the crack front, and  $x_1$  points in the direction of crack growth. The position of the crack tip is defined by  $x_1 = a(t)$ . A moving coordinate system,  $x, y, z$  is centered at the crack tip, with its axes parallel to the  $x_1, x_2$  and  $x_3$  axes. Relative to the moving coordinate system we also define polar coordinates  $r, \theta$ , with  $\theta = 0$  coinciding with the positive  $x$  direction.

In the moving coordinate system the equilibrium equations are

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} = 0, \quad \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0 \quad (1a, b)$$

We consider a state of generalized plane stress, hence  $\sigma_z$ ,  $\sigma_{xz}$  and  $\sigma_{yz}$  vanish identically. The Huber-Mises yield criterion may then be written

$$\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\tau_{xy}^2 = 3k^2 \quad (2)$$

where  $k$  is the yield stress in pure shear. The strain rates are

$$\dot{\epsilon}_x = \frac{\partial \dot{u}}{\partial x}, \quad \dot{\epsilon}_y = \frac{\partial \dot{v}}{\partial y}, \quad \dot{\epsilon}_{xy} = \frac{1}{2} \left( \frac{\partial \dot{u}}{\partial y} + \frac{\partial \dot{v}}{\partial x} \right) \quad (3a, b, c)$$

In the moving coordinate system the material time derivative is

$$(\dot{\phantom{x}}) = \partial_t - \dot{a} \partial_x \quad (4)$$

where  $\dot{a} = da/dt$  is the speed of the crack tip. The strain rates are related to the stresses and stress rates by

$$\frac{\partial \dot{u}}{\partial x} = \frac{1}{E}(\dot{\sigma}_x - \nu \dot{\sigma}_y) + \frac{1}{3} \dot{\Lambda}(2\sigma_x - \sigma_y) \quad (5)$$

$$\frac{\partial \dot{v}}{\partial y} = \frac{1}{E}(\dot{\sigma}_y - \nu \dot{\sigma}_x) + \frac{1}{3} \dot{\Lambda}(2\sigma_y - \sigma_x) \quad (6)$$

$$\frac{1}{2}\left(\frac{\partial \dot{u}}{\partial y} + \frac{\partial \dot{v}}{\partial x}\right) = \frac{1+\nu}{E} \dot{\tau}_{xy} + \dot{\Lambda} \tau_{xy} \quad (7)$$

where  $E$  and  $\nu$  are Young's modulus and Poisson's ratio, respectively and  $\dot{\Lambda}$  is a positive function of time and the spatial coordinates.

#### Solution along the Crack Line

In this paper we are interested in solutions along the crack line  $y = 0$ ,  $0 < x \leq x_p$ , where  $x = x_p$  defines the elastic-plastic boundary. Such solutions can be obtained by considering expansions with respect to  $y$  in the region  $y/x \ll 1$ :

$$\sigma_x = p_0(x, t) + p_2(x, t)y^2 + p_4(x, t)y^4 + \dots \quad (8)$$

$$\sigma_y = q_0(x, t) + q_2(x, t)y^2 + q_4(x, t)y^4 + \dots \quad (9)$$

$$\tau_{xy} = s_1(x, t)y + s_3(x, t)y^3 + \dots \quad (10)$$

$$\dot{u} = \dot{u}_0(x, t) + \dot{u}_2(x, t)y^2 + \dots \quad (11)$$

$$\dot{v} = \dot{v}_1(x, t)y + \dot{v}_3(x, t)y^3 + \dots \quad (12)$$

$$\dot{\Lambda} = \dot{\Lambda}_0(x, t) + \dot{\Lambda}_2(x, t)y^2 + \dots \quad (13)$$

Here we have taken into account that  $\sigma_x$ ,  $\sigma_y$ ,  $u$  and  $\dot{\Lambda}$  are symmetric with respect to  $y = 0$ , while  $\sigma_{xy}$ ,  $\dot{v}$  are antisymmetric. Substitution of



(8)-(10) into (1a,b) and collecting terms of the same order in  $y$  yields

$$\frac{\partial p_0}{\partial x} + s_1 = 0 \quad (14)$$

$$\frac{\partial p_2}{\partial x} + 3s_3 = 0 \quad (15)$$

$$\frac{\partial s_1}{\partial x} + 2q_2 = 0 \quad (16)$$

$$\frac{\partial s_3}{\partial x} + 4q_4 = 0 \quad (17)$$

Substitution of (8)-(10) into the yield condition (2) yields by the same procedure

$$p_0^2 + q_0^2 - p_0 q_0 = 3k^2 \quad (18)$$

$$(2p_0 - q_0)p_2 + (2q_0 - p_0)q_2 + 3s_1^2 = 0 \quad (19)$$

$$p_2^2 + (2p_0 - q_0)p_4 + q_2^2 + (2q_0 - p_0)q_4 - p_2 q_2 + 6s_1 s_3 = 0 \quad (20)$$

In the same manner we obtain by using (8)-(13) in (5)-(7)

$$\frac{\partial \dot{u}_0}{\partial x} = \frac{1}{E} \left( \frac{\partial p_0}{\partial t} - \nu \frac{\partial q_0}{\partial t} \right) - \frac{\dot{a}}{E} \left( \frac{\partial p_0}{\partial x} - \nu \frac{\partial q_0}{\partial x} \right) + \frac{\dot{\lambda}_0}{3} (2p_0 - q_0) \quad (21)$$

$$\frac{\partial \dot{u}_2}{\partial x} = \frac{1}{E} \left( \frac{\partial p_2}{\partial t} - \nu \frac{\partial q_2}{\partial t} \right) - \frac{\dot{a}}{E} \left( \frac{\partial p_2}{\partial x} - \nu \frac{\partial q_2}{\partial x} \right) + \frac{\dot{\lambda}_0}{3} (2p_2 - q_2) + \frac{\dot{\lambda}_2}{3} (2p_0 - q_0) \quad (22)$$

$$\dot{v}_1 = \frac{1}{E} \left( \frac{\partial q_0}{\partial t} - \nu \frac{\partial p_0}{\partial t} \right) - \frac{\dot{a}}{E} \left( \frac{\partial q_0}{\partial x} - \nu \frac{\partial p_0}{\partial x} \right) + \frac{\dot{\lambda}_0}{3} (2q_0 - p_0) \quad (23)$$

$$3\dot{v}_3 = \frac{1}{E} \left( \frac{\partial q_2}{\partial t} - \nu \frac{\partial p_2}{\partial t} \right) - \frac{\dot{a}}{E} \left( \frac{\partial q_2}{\partial x} - \nu \frac{\partial p_2}{\partial x} \right) + \frac{1}{3} \dot{\lambda}_0 (2q_2 - p_2) + \frac{1}{3} \dot{\lambda}_2 (2q_0 - p_0) \quad (24)$$

$$\dot{u}_2 + \frac{1}{2} \frac{\partial \dot{v}_1}{\partial x} = \frac{1+\nu}{E} \left( \frac{\partial s_1}{\partial t} - \dot{a} \frac{\partial s_1}{\partial x} \right) + \dot{\lambda}_0 s_1 \quad (25)$$

where (4) has also been used.

At this stage we have 14 unknowns and 12 equations. It turns out that this system of equations can be solved provided that one assumption is made. We assume that  $\sigma_y$  is constant on the crack line in the plastic loading zone. Hence, by the use of Eq.(9):

$$q_0 = \text{constant} \quad (26)$$

It will be shown later that the assumption is consistent with the stress components for a centered fan field which have been considered by Hutchinson [4], and which are:

$$\sigma_x = k \cos^3 \theta, \quad \sigma_y = k(2 \cos^3 \theta + 3 \sin^2 \theta \cos \theta), \quad \sigma_{xy} = -k \sin^3 \theta \quad (27a,b,c)$$

If  $q_0 = \text{constant}$ , it follows from Eq.(18) that  $p_0 = \text{constant}$ . On the basis of  $p_0 = \text{constant}$ , it follows from Eq.(14) that  $s_1 = 0$ , and subsequently from Eq.(16) that  $q_2 = 0$ . Substitution of these results in Eq.(19) yields  $q_0 = 2p_0$ , and Eq.(18) then gives  $p_0 = k$  and  $q_0 = 2k$ . Equation (20) subsequently yields  $q_4 = -p_2^2/3k$ . Substitution of the latter result in Eq.(17) and then in Eq.(15) yields

$$\frac{\partial^2 p_2}{\partial x^2} + \frac{4}{k} p_2^2 = 0 \quad (28)$$

The solution to Eq.(28) which satisfies the condition that the stresses are multivalued at the crack tip is

$$p_2 = -\frac{3}{2} \frac{k}{x^2} \quad (29)$$

In summary, it has been shown that the equilibrium equations and the yield condition are satisfied by

$$\sigma_x = k \left[ 1 - \frac{3}{2} \left( \frac{y}{x} \right)^2 \right] + o \left( \frac{y}{x} \right)^4 \quad (30)$$

$$\sigma_y = 2k + o \left( \frac{y}{x} \right)^4 \quad (31)$$

$$\tau_{xy} = -k \left( \frac{y}{x} \right)^3 + o \left( \frac{y}{x} \right)^5 \quad (32)$$

It is noted that Eqs.(30)-(31) are indeed expansions with respect to  $\theta = y/x$  of Eqs.(27a,b,c).

In the next step we substitute Eqs.(30)-(32) into (21)-(25) and collect terms of the same order. The result is

$$\frac{\partial \dot{u}_0}{\partial x} = 0, \quad \frac{\partial \dot{u}_2}{\partial x} = -3 \frac{k}{E} \frac{\dot{a}}{x^3} - \frac{1}{x^2} \dot{\Lambda}_0 k \quad (33)$$

$$\dot{v}_1 = \dot{\Lambda}_0 k \quad (34)$$

$$2\dot{u}_2 + \frac{\partial \dot{v}_1}{\partial x} = 0 \quad (35)$$

By combining (33), (34) and (35) we obtain

$$\frac{1}{2} \frac{\partial^2 \dot{v}_1}{\partial x^2} - \frac{\dot{v}_1}{x^2} = 3 \frac{k}{E} \frac{\dot{a}}{x^3} \quad (36)$$

The general solution to Eq.(36) is

$$\dot{v}_1 = \frac{k}{E} \left\{ -2 \frac{\dot{a}}{x} \ln \left( \frac{x}{x_p} \right) + \frac{B(t)}{x} + C(t)x^2 \right\}, \quad (37)$$

where  $x_p$  defines the x-coordinate of the elastic-plastic boundary on the crack line. The functions  $B(t)$  and  $C(t)$  can be obtained from continuity conditions at the elastic-plastic boundary.

For small values of  $\theta$  (i.e.,  $y/x \ll 1$ ) the field in the plastic loading zone will be matched at the elastic-plastic boundary to the dominant terms of a corresponding elastic field. For the elastic field we do, however, not take the field for a crack, but rather that for a notch with  $\frac{1}{2}\rho$  as radius of curvature at its tip. In polar coordinates  $R, \psi$ , the appropriate Mode-I stress fields are given by Creager and Paris [ 5] as

$$\sigma_x = \left( \frac{1}{2\pi R} \right)^{\frac{1}{2}} K_I \left\{ \cos \frac{1}{2}\psi \left[ 1 - \sin \frac{1}{2}\psi \sin \frac{3}{2}\psi \right] - \frac{\rho}{2R} \cos \frac{3}{2}\psi \right\} \quad (38)$$

$$\sigma_y = \left( \frac{1}{2\pi R} \right)^{\frac{1}{2}} K_I \left\{ \cos \frac{1}{2}\psi \left[ 1 + \sin \frac{1}{2}\psi \sin \frac{3}{2}\psi \right] + \frac{\rho}{2R} \cos \frac{3}{2}\psi \right\} \quad (39)$$

$$\tau_{xy} = \left( \frac{1}{2\pi R} \right)^{\frac{1}{2}} K_I \left\{ \sin \frac{1}{2}\psi \cos \frac{1}{2}\psi \cos \frac{3}{2}\psi - \frac{\rho}{2R} \sin \frac{3}{2}\psi \right\} \quad (40)$$

Note that the tip of the notch, which is not the tip of the crack nor the elastic-plastic boundary, is a distance  $\frac{1}{2}\rho$  from the origin E, as shown in Fig. 1. The center of the elastic field E, whose position is defined by  $x_1 = e(t)$ ,  $y_1 = 0$ , is located in between the crack tip and the elastic-plastic boundary defined by  $x = x_p(t)$ . For generalized plane stress, the displacements corresponding to (38)-(40) are

$$u = \left( \frac{R}{2\pi} \right)^{\frac{1}{2}} \frac{1}{2\mu} K_I \left\{ \cos \frac{1}{2}\psi \left[ \kappa - 1 + 2\sin^2 \frac{1}{2}\psi \right] + \frac{\rho}{R} \cos \frac{1}{2}\psi \right\} \quad (41)$$

$$v = \left(\frac{R}{2\pi}\right)^{\frac{1}{2}} \frac{1}{2\mu} K_I \left\{ \sin \frac{1}{2}\psi \left[ \kappa + 1 - 2\cos^2 \frac{1}{2}\psi \right] + \frac{\rho}{R} \sin \frac{1}{2}\psi \right\} \quad (42)$$

where  $\kappa = (3-\nu)/(1+\nu)$ .

From the condition that the elastic field should just reach the yield condition at the elastic-plastic boundary, we obtain by the use of (2) and (38)-(40)

$$\left[ \left( \frac{1}{2\pi R_p} \right)^{\frac{1}{2}} K_I \right]^2 \left[ 1 + 3 \left( \frac{\rho}{2R_p} \right)^2 \right] = 3k^2 \quad (43)$$

where  $R = R_p$  at the elastic-plastic boundary, at least for small values of  $y$ . Another condition is that  $\sigma_x$  should be continuous at the elastic-plastic boundary on  $y=0$ . By the use of (30) and (38) we find

$$k = \left( \frac{1}{2\pi R_p} \right)^{\frac{1}{2}} K_I \left( 1 - \frac{\rho}{2R_p} \right) \quad (44)$$

From (43) and (44) it follows that

$$\frac{\rho}{R_p} = \frac{2}{3} \quad (45)$$

Hence (44) yields

$$\left( \frac{1}{2\pi R_p} \right)^{\frac{1}{2}} K_I = \frac{3}{2}k \quad (46)$$

Equations (43) and (46) show why we have taken elastic fields for a notch rather than for a crack. For an elastic crack-tip field the conditions of reaching the yield condition at the elastic-plastic boundary would conflict with the condition of continuity of  $\sigma_x$ , as can be checked by setting  $\rho \equiv 0$  in (43) and (44).

Further details of the matching procedure have been given by Achenbach and Dunayevsky [3]. The relevant results are

$$x_E = (1-\gamma)x_p, \text{ and thus } R_p = \gamma x_p, \quad (47,a,b)$$

where  $\gamma = 1/\sqrt{2}$ .

$$B(t) = B_1 \dot{a}(t) + B_2 \dot{x}_p(t) \quad (48)$$

$$C(t) = [C_1 \dot{a}(t) + C_2 \dot{x}_p(t)]/[x_p(t)]^3, \quad (49)$$

where

$$B_1 = \frac{1}{32} \frac{1}{\gamma^2} [\kappa+5 + 4\gamma(\kappa+1)](E/\mu) - \frac{2}{3} \quad (50)$$

$$B_2 = \frac{1}{32} \frac{1}{\gamma^2} [\kappa+5 + 2\gamma(\kappa+1)](E/\mu) \quad (51)$$

$$C_1 = \frac{1}{32} \frac{1}{\gamma^2} [-(\kappa+5) + 2\gamma(\kappa+1)](E/\mu) + \frac{2}{3} \quad (52)$$

$$C_2 = \frac{1}{32} \frac{1}{\gamma^2} [-(\kappa+5) + 4\gamma(\kappa+1)](E/\mu) \quad (53)$$

#### The Strain on the Crack Line

In the plane of the crack we have  $\epsilon_y = v_1$ . At the elastic plastic boundary (42) yields for small  $y$

$$(\epsilon_y)_{PB} = \frac{3}{8} \frac{k}{\mu} \left(\kappa - \frac{1}{3}\right) = \frac{2-\nu}{E} k \quad (54)$$

where (46), (47) and  $\theta \sim y/x$  have been used. In the stationary coordinate system, (37) can now be integrated to yield the total strain for  $t \geq t_p$  as

$$\epsilon_y(x_1, t) = (\epsilon_y)_{PB} + \int_{t_p}^t \dot{v}_1(x_1, s) ds \quad (55)$$

where  $\dot{v}_1(x_1, s)$  is obtained from (37) by using the relation  $x = x_1 - a(t)$ . In (55),  $t_p$  is the time that the elastic-plastic boundary arrives at position  $x_1$ . Thus, for the propagating crack tip,  $t_p$  follows from the equation

$$a(t_p) + x_p(t_p) = x_1, \quad (56)$$

Here  $x_p(t_p)$  is obtained from the stress intensity factor by using (46) and (47b):

$$x_p(t) = \frac{2\sqrt{2}}{9} \frac{1}{\pi} [K_I(t)/k]^2 \quad (57)$$

Equation (37) is also valid for a stationary crack. By setting  $\dot{a} \equiv 0$  we obtain from (37) for  $t \geq t_p$

$$\dot{v}_1^{sc}(\Delta, t) = \frac{k}{E} [B_2/\Delta + C_2\Delta^2/x_p^3(t)] \dot{x}_p(t), \quad (58)$$

where (48) and (49) have been used and  $\Delta$  is a distance ahead of the crack tip. The arrival time of the elastic-plastic boundary follows from (56) as

$$a_o + x_p(t_p) = x_1 \quad (59)$$

Let us consider the case that loading starts at time  $t = 0$ , but that the crack tip does not start to propagate until time  $t = t_s$ . For a position  $x_1$  which is inside the plastic zone at time  $t = t_s$ , we find

$$\epsilon_y(x_1, t_e) = (\epsilon_y)_{PB} + \int_{t_p}^t \dot{v}_1^{sc}(x_1, t) ds + \int_{t_s}^{t_e} \dot{v}_1(x_1, s) ds \quad (60)$$

Here  $t_p$  follows from (59), and  $t_e$  is the time that the crack tip arrives at a small distance  $\Delta$  from the position  $x_1$  that is being observed. We have

$$x_1 - \Delta = a(t_e) \quad (61)$$

For a position  $x_1$  which is outside the plastic zone at time  $t = t_s$ ,

$$x_1 > a_0 + x_p(t_s), \quad (62)$$

the expression (55) holds, where  $t_p$  is now defined by (56).

Equations (55) and (60) can be manipulated to yield the singular parts of the strain at  $x_1 = a(t)$ , plus a bounded integral. The result can be found in Ref.[3]. In the present paper the integrals (55) and (60) have been evaluated numerically.

The result simplifies considerably for the case that all fields are assumed to be time-invariant to an observer traveling with the crack tip. This is the steady-state case when  $\epsilon_y$  depends on  $x = x_1 - a(t)$  only. Now we have that  $\dot{a} = \text{constant} = c_F$ ,  $\dot{x}_p = 0$ , and  $(\dot{\phantom{x}}) = -c_F d/dx$ . The solution to Eq.(37) becomes

$$\frac{dv_1}{dx} = \frac{k}{E} \left\{ \frac{2}{x} \ln\left(\frac{x}{x_p}\right) - \frac{B}{x} - \frac{C_1 x^2}{x_p^3} \right\} \quad (63)$$



where  $B_1$ ,  $C_1$  are defined by (50) and (52), and  $x_p = \text{constant}$ .

Equation (56) may be integrated to yield

$$\epsilon_y(x) = (\epsilon_y)_{PB} + \frac{k}{E} \left\{ \left[ \ln\left(\frac{x}{x_p}\right) \right]^2 - B_1 \ln\left(\frac{x}{x_p}\right) - \frac{1}{3} C_1 \left[ \left(\frac{x}{x_p}\right)^3 - 1 \right] \right\} \quad (64)$$

### Numerical Results

For a given external load, we presumably know  $K_I$  in terms of the crack length  $a(t)$ . A relation between  $x_p(t)$  and  $a(t)$  can subsequently be obtained by the use of (57). Hence, in principle,  $a(t)$  is the only unknown quantity in (55). An equation for  $a(t)$  and  $\dot{a}(t)$  can, for example, be obtained from (55) by the use of the critical strain criterion for crack propagation. This criterion stipulates that crack growth will proceed when a critical strain level  $\epsilon_{cr}$  is maintained for  $\epsilon_y$  in the plane of the crack at a characteristic distance  $\Delta$  ahead of the crack tip. It appears, however, that it will be very difficult to solve  $a(t)$  and  $\dot{a}(t)$  from the integral equation that can be extracted from (55).

In the examples that are considered here, we consider an inverse problem, that is, we prescribe the increasing crack length  $a(t)$  and the variation of the distant tensile stresses  $\sigma(t)$ , and we use (55) and (60) to compute the strain at a small distance  $\Delta$  ahead of the crack tip.

Numerical results have been obtained for a material with the following mechanical properties, which are comparable to those of CrMnSiNi Steel:

Young's modulus:  $E = 2.06 \times 10^{11} \text{ N/m}^2$

Poisson's ratio:  $\nu = 0.3$

Yield stress in shear:  $k = 8.13 \times 10^8 \text{ N/m}^2$

Plane stress fracture toughness:  $K_C = 16.7 \times 10^7 \text{ N/m}^2$

The geometry considered was an edge crack of initial length

$a_0 = 50\text{mm}$

in a half-plane. The half-plane was subjected to distant tensile stresses of magnitude  $\sigma(t)$ . The relevant stress intensity factor was taken as

$$K_I = 1.1215 [\pi a(t)]^{1/2} \sigma(t) \quad (65)$$

The first example attempts to consider a case where the stress intensity factor varies in such a manner that a steady-state situation can be established. We choose

$$a(t) = a_0 \quad t \leq t_s \quad (66a)$$

$$= a_0 + a_1(t-t_s) + a_1\{\exp[-(t-t_s)] - 1\} \quad t \geq t_s \quad (66b)$$

and

$$\sigma(t) = (t/t_s)\sigma_0 \quad t \leq t_s \quad (67a)$$

$$= \left[ \frac{a_0}{a(t)} \left\{ 1 + \left( \frac{a_c}{a_0} - 1 \right) \frac{t-t_s}{t_c-t_s} \right\} \right]^{1/2} \sigma_0 \quad t_s \leq t \leq t_c \quad (67b)$$

$$= \left[ \frac{a_c}{a(t)} \right]^{1/2} \sigma_0 \quad t_c \leq t \quad (67c)$$

Here  $\sigma_0$  was taken as the time that  $K_I$  reaches the value of the fracture toughness

$$\sigma_0 = K_C / (1.1215 \sqrt{\pi a_0}) \quad (68)$$

while  $t_c$  is the time that the crack tip arrives at a distance  $\Delta$  from the position of the elastic-plastic boundary at time  $t = t_s$ . Thus  $t_c$  can be computed from

$$a(t_c) + \Delta = a_0 + x_p(t_s) \quad (69)$$

The length  $a_c$  was taken as  $a_c = 63.9\text{mm}$ , and the time  $t_s$  was taken as  $t_s = 30\text{s}$ . It follows from (67c) and (65) that  $K_I$  remains constant for  $t > t_c$ . The crack-tip speed  $\dot{a}(t)$  and the distant tensile stress  $\sigma(t)$  have been plotted in Fig. 2a and Fig. 2b, respectively.

The strain  $\epsilon_y$  has been plotted in Fig. 3. The upper curve represents the strain at a fixed position  $\Delta = 1\text{mm}$  ahead of the crack tip. For  $t < t_s = 30\text{s}$ , this is a fixed position. For  $t > t_s$  the position moves with the crack tip. The curves numbered 2-10 represent the strains for specific fixed material points. Points 2, 3 and 4 were located inside the plastic zone at time  $t = t_s$ , and the corresponding strains were computed by Eq.(60 with upper limit  $t \leq t_e$ . Points 5-10 were outside the plastic zone at time  $t = t_s$ , and for these points the strains were computed by Eq.(55). Curves 2-10 all end at a time  $t_e$  at which the crack tip is a distance  $\Delta$  from the material point. Time  $t_e$  is computed from Eq.(61). It is noted that the strain quickly approaches an apparent steady-state value, which is just equal to the steady-state value that can be computed from Eq.(64). Thus, for the distant stress given by Eq.(67), stable crack propagation at a constant strain  $\epsilon_y$  is established quickly.

For the second example we choose

$$\sigma(t) = \left(\frac{t}{t_s}\right)\sigma_0 \quad \text{for } t \leq t_s \quad (70a)$$

$$= \sigma_0 \quad \text{for } t \geq t_s, \quad (70b)$$

where  $\sigma_0$  is defined by Eq.(68). The crack starts to propagate with

constant velocity  $c_F$  at time  $t = t_s$ :

$$a(t) = a_o \quad \text{for } t \leq t_s \quad (71a)$$

$$= a_o + c_F(t - t_s) \quad \text{for } t \geq t_s \quad (71b)$$

For three crack-tip speeds, the strains at a location which remains at a fixed distance of  $\Delta = 3\text{mm}$  ahead of the crack tip are shown in Fig. 4. Thus, the location where the strain is computed is stationary for  $t \leq t_s$ , and it moves with the crack-tip speed  $c_F$  for  $t > t_s$ . The strains increase with time, and the rate of change depends significantly on the crack tip speed.

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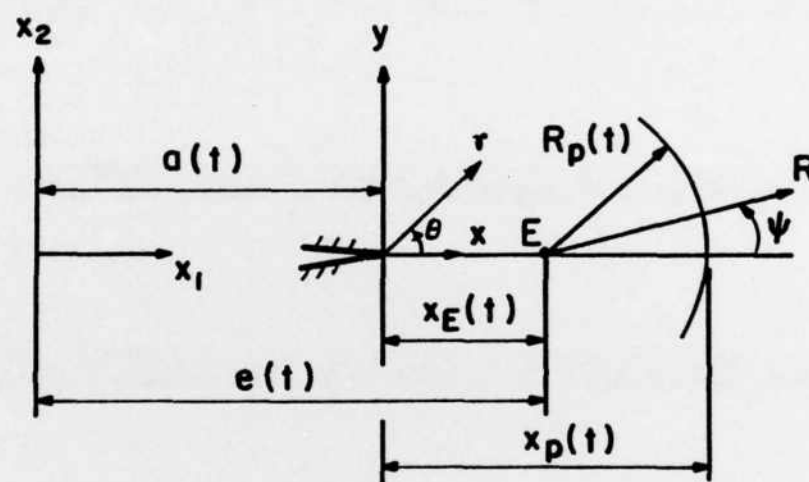


Fig. 1: Center of elastic field  $E$ , for slender notch of tip radius  $\rho$

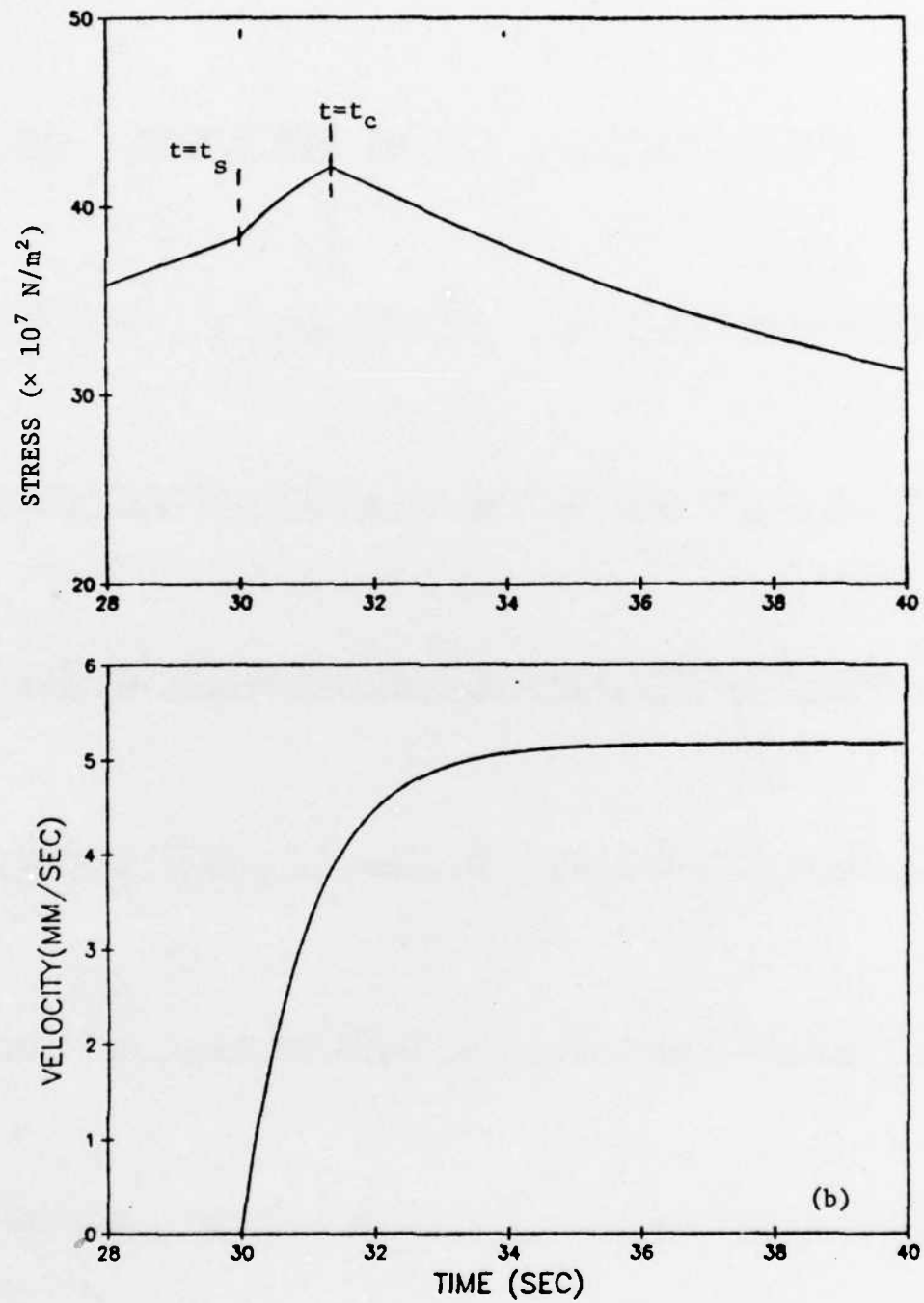


Fig. 2: (a) Stress field  $\sigma(t)$  according to (67 a-c), and  
(b) Crack-tip speed from Eq.(66b).



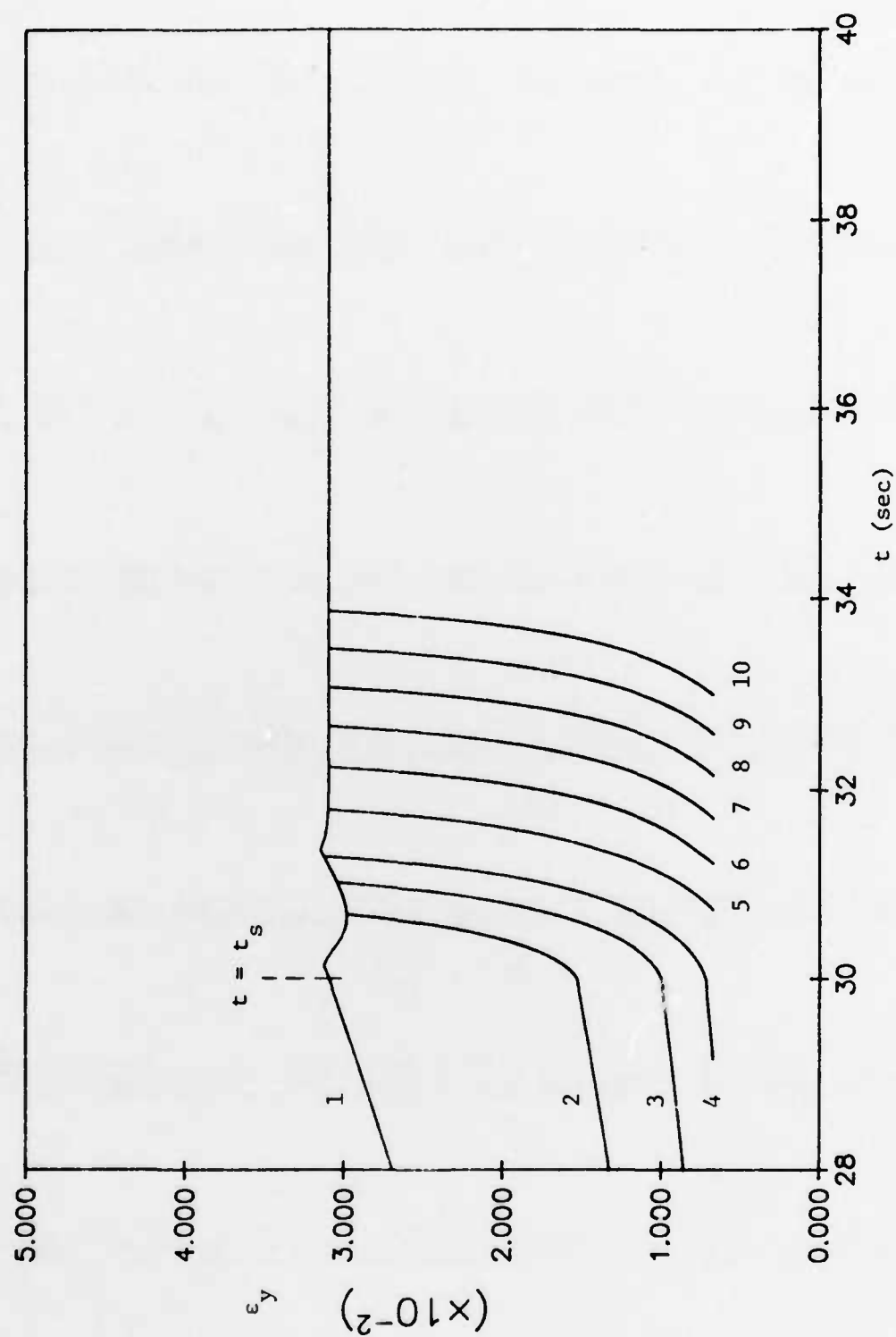


Fig. 3: Strain  $\epsilon_y$  versus time; curve 1: at fixed distance  $\Delta$  ahead of the stationary ( $t < t_s$ ) and the moving crack tip ( $t \geq t_s$ ); curves 2-10: at fixed material points;  $\Delta \approx 1\text{mm}$ .

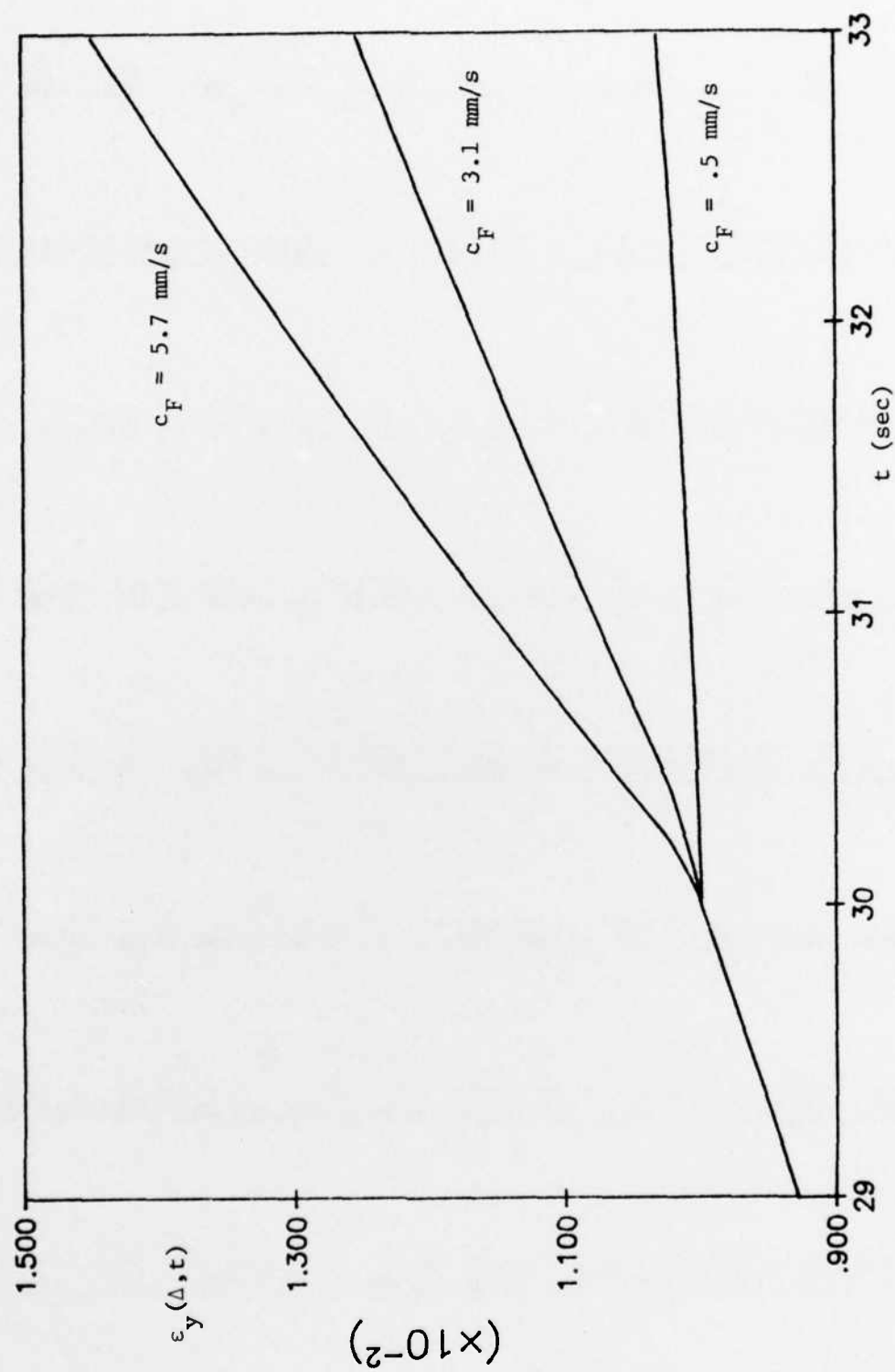


Fig. 4: Strain  $\epsilon_y$  at fixed distance  $\Delta$  ahead of crack tip, for ramp stress defined by (70a,b) and constant crack tip speed, see (71a,b);  $\Delta = 3\text{mm}$ .

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Mode-I crack growth in an elastic perfectly-plastic material under conditions of generalized plane stress has been investigated. In the plastic loading zone, near the plane of the crack, the stresses and strains have been expanded in powers of the distance, $y$ , to the crack line. Substitution of the expansions in the equilibrium equations, the yield condition and the constitutive equations yields a system of simple ordinary differential equations for the coefficients of the expansions. This system is solvable if it is assumed that the cleavage stress is uniform on the crack line. By matching the relevant stress components and		

particle velocities to the dominant terms of appropriate elastic fields at the elastic-plastic boundary, a complete solution has been obtained for  $\epsilon_y$  in the plane of the crack. The solution depends on crack-line position and time, and applies from the propagating crack tip up to the moving elastic-plastic boundary. Numerical results are presented for the edge crack geometry.

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